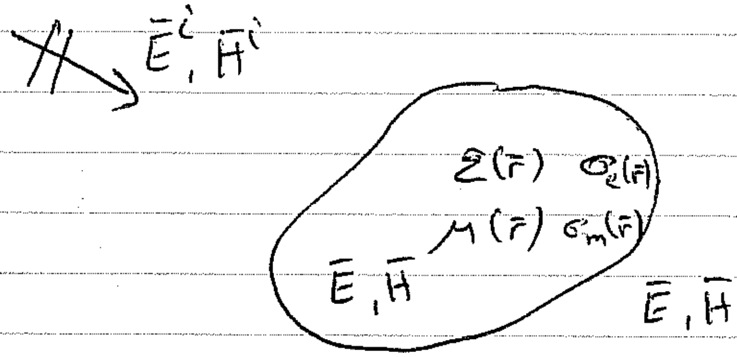
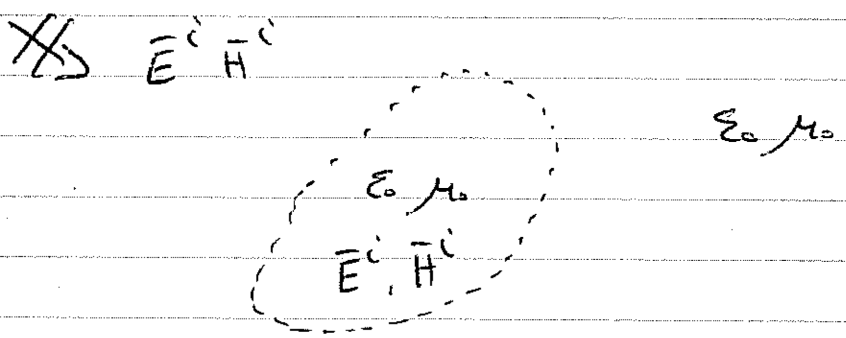
Scattered Field Formulation of FDTD

Suppose we have in homogeneous material from which we want to calculate the scattered field.



(problem of interest)

The <u>incident field</u> is defined as the electromagnetic field produced by the sources, say some antenna outside our domain of interest, in the absence of the inhomogeneous scatterer.



In our computational domain, these fields satisfy

$$\begin{cases} \bar{\nabla} \times \bar{H}^i = \varepsilon_0 \, \partial_t \bar{E}^i \\ \bar{\nabla} \times \bar{E}^i = -\mu_0 \, \partial_t \bar{H}^i \end{cases} \qquad (1)$$

In the original problem, we call the fields $\bar{E}, \bar{H}$ the <u>total field</u>. The total fields satisfy

$$\begin{cases} \bar{\nabla} \times \bar{H} = \varepsilon(\bar{r}) \partial_t \bar{E} + \sigma_e(\bar{r}) \bar{E} \\ \bar{\nabla} \times \bar{E} = -\mu(\bar{r}) \partial_t \bar{H} - \sigma_m(\bar{r}) \bar{H} \end{cases} \quad (2)$$

where $\varepsilon(\bar{r})$, $\mu(\bar{r})$, $\sigma_e(\bar{r})$ and $\sigma_m(\bar{r})$ are the inhomogeneous permittivity, permeability, electric conductivity, and magnetic conductivity.

We now define the <u>scattered field</u> in terms of these two fields as:

$$\bar{E}^s \triangleq \bar{E} - \bar{E}^i \qquad \bar{H}^s \triangleq \bar{H} - \bar{H}^i \quad (3)$$

We can find the equations that $\bar{E}^s$ and $\bar{H}^s$ satisfy by plugging in $\bar{E} = \bar{E}^s + \bar{E}^i$ and $\bar{H} = \bar{H}^s + \bar{H}^i$ into (2) and then using (1):

$$\bar{\nabla} \times \bar{H}^s + \bar{\nabla} \times \bar{H}^i = \varepsilon(\bar{r}) \partial_t \bar{E}^s + \varepsilon(\bar{r}) \partial_t \bar{E}^i + \sigma_e(\bar{r}) \bar{E}^s + \sigma_e(\bar{r}) \bar{E}^i$$
$$\bar{\nabla} \times \bar{E}^s + \bar{\nabla} \times \bar{E}^s = -\mu(\bar{r}) \partial_t \bar{H}^s - \mu(\bar{r}) \partial_t \bar{H}^s - \sigma_m(\bar{r}) \bar{H}^s - \sigma_m(\bar{r}) \bar{H}^i$$

$$\bar{\nabla} \times \bar{H}^s + \varepsilon_0 \partial_t \bar{E}^i = \varepsilon(\bar{r}) \partial_t \bar{E}^s + \varepsilon(\bar{r}) \partial_t \bar{E}^i + \sigma_e(\bar{r}) \bar{E}^s + \sigma_e(\bar{r}) \bar{E}^i$$
$$\bar{\nabla} \times \bar{E}^s - \mu_0 \partial_t \bar{H}^i = -\mu(\bar{r}) \partial_t \bar{H}^s - \mu(\bar{r}) \partial_t \bar{H}^s - \sigma_m(\bar{r}) \bar{H}^s - \sigma_m(\bar{r}) \bar{H}^i$$

$$\boxed{\begin{aligned} \varepsilon(\bar r)\,\partial_t \bar E^s + \sigma_e(\bar r)\,\bar E^s &= \bar\nabla\times\bar H^s + (\varepsilon_o - \varepsilon(\bar r))\,\partial_t \bar E^i - \sigma_e(\bar r)\,\bar E^i \\ \mu(\bar r)\,\partial_t \bar H^s + \sigma_m(\bar r)\,\bar H^s &= -\bar\nabla\times\bar E^s + (\mu_o - \mu(\bar r))\,\partial_t \bar H^i - \sigma_m(\bar r)\,\bar H^i \end{aligned}}$$

the terms:

$$\bar J_{eq} \triangleq (\varepsilon(\bar r) - \varepsilon_o)\,\partial_t \bar E^i + \sigma_e(\bar r)\,\bar E^i$$

$$\bar M_{eq} \triangleq (\mu(\bar r) - \mu_o)\,\partial_t \bar H^i + \sigma_m(\bar r)\,\bar E^i$$

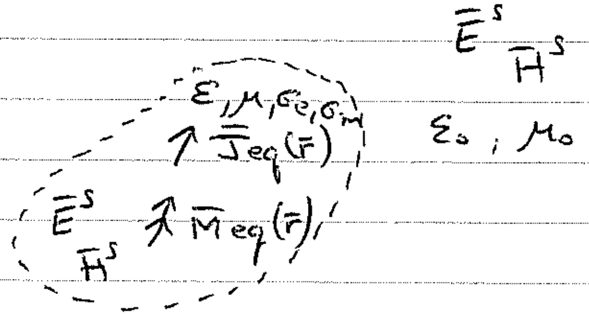
are equivalent sources, the strength of which is proportional to the contrasts

$$\chi_\varepsilon \triangleq (\varepsilon(\bar r) - \varepsilon_o)$$
$$\chi_\mu \triangleq (\mu(\bar r) - \mu_o)$$

and the time rate of change of the incident field, and the conductivities and the incident field.

Note that for a particular problem, we will know, or approximate the incident field, $\bar E^i$ and $\bar H^i$, and we will know the contrast functions $\chi_\varepsilon(\bar r)$ and $\chi_\mu(\bar r)$.

Pictorially we have



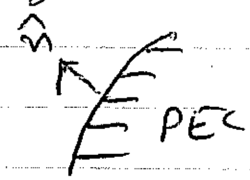
So the scattered fields $\bar{E}^s, \bar{H}^s$ are produced by equivalent sources

$$\boxed{\begin{aligned} \varepsilon(\bar{r})\,\partial_t \bar{E}^s + \sigma_e(\bar{r})\,\bar{E}^s &= \bar{\nabla}\times\bar{H}^s - \bar{J}_{eq} \\ \mu(\bar{r})\,\partial_t \bar{H}^s + \sigma_m(\bar{r})\,\bar{H}^s &= -\bar{\nabla}\times\bar{E}^s - \bar{M}_{eq} \end{aligned}}$$

<u>PEC Boundary</u>

at a perfec-electric-conducting BC

we must have $\hat{n}\times\bar{E} = 0$



but $\bar{E} = \bar{E}^i + \bar{E}^s$

$\therefore$ we must impose $\hat{n}\times\bar{E}^s = -\hat{n}\times\bar{E}^i$